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RESISTANCE OF A CIRCULATING FLUIDIZED BED

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Available experimental data on the resistance of the riser of a circulating fluidized bed under different conditions of control over its operation are generalized within the framework of similarity theory. Dependences for calculating the height of the fluidized bed formed in the lower part of the riser are obtained based on an analysis of the components of the resistance of this bed.

Technologies employing a circulating fluidized bed (CFB) are widely adopted in industry, especially in power engineering. Due to the comparatively short period of time during which a CFB has been investigated, its main characteristics have not been studied sufficiently as yet. Among these characteristics is the hydraulic resistance of a CFB, which is of importance in practice. Experimental data on the main component of this quantity − the resistance of the CFB riser (Δp_r) − are available in the literature [1–5]. Due to the absence of generalized computational dependences these results are of limited practical utility. By virtue of this fact, the present work is aimed at obtaining generalized dependences for calculation of Δp_r using available data on the resistance of a CFB riser and the similarity theory of transport processes in CFB [6, 7].

We analyze the resistance of a CFB separately for a bed operating according the "chemical reactor" scheme (a system with forced circulation of the particles in which the circulating particle flux J_s is an independent parameter) and for a "furnace"-type CFB − a freely circulating CFB where another quantity is the independent parameter that determines the number of particles in the riser (most often this is the resistance of the previously determined lower part of the bed Δp_{ref} [8]); in this case, the particle flux J_s is not controlled.*)

"Chemical Reactor." In [7], for the expression for an arbitrary dimensionless hydrodynamic characteristic of a CFB it was obtained that

$$\Gamma' = f(\bar{J}_s, \mathrm{Fr}_t, h/H, H/D) . \quad (1)$$

As applied to the resistance of the riser, Eq. (1) takes the form

$$\frac{\Delta p_r}{J_s (u - u_t)} = f(\bar{J}_s, \mathrm{Fr}_t, H/D) . \quad (2)$$

Equation (2) was used in generalization of the test data mentioned above [1-5]. For a function f of standard power form the following empirical dependence is established:

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*) The difference in these types of CFBs is not just formal. They also differ in the value of *H/D*: for a "chemical reactor" *H/D* = 22−75, for a "furnace" *H/D* = 5−10 [9]. This causes the presence of a certain difference in the functional dependences for these CFBs (see below).

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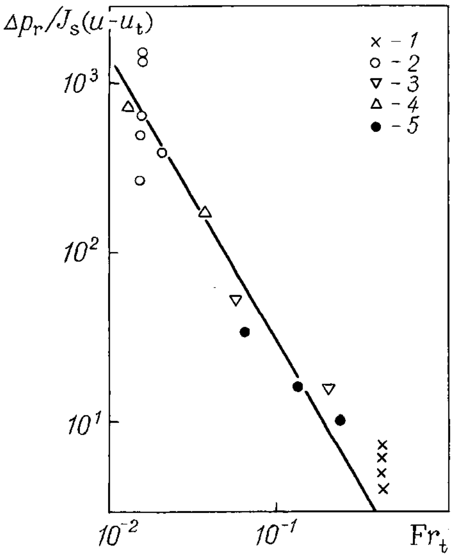

Fig. 1. Resistance of a circulating fluidized bed of the "chemical reactor" type: 1) [1]; 2) [2]; 3) [3]; 4) [4]; 5) [5].

$$\frac{\Delta p_r}{J_s(u-u_t)} = 0.6\mathrm{Fr}_t^{-1.68}, \tag{3}$$

which is verified for $\mathrm{Fr}_t = 1.5\cdot10^{-2}$–$4.2\cdot10^{-1}$. A comparison of experimental values of Δp_r and ones calculated by (3) is shown in Fig. 1. The root-mean-square scatter of the experimental points is 18%.

Taking into consideration the actual structure of a CFB [1, 8], we can represent the quantity Δp_r as a sum of three terms:

$$\Delta p_r = \Delta p_{fb} + \Delta p_{tr} + \Delta p_{fr}. \tag{4}$$

As was shown by the estimates made in [1], the contribution of the friction of the gas and the particles against the riser walls can be neglected. Representing Δp_{fb} in the form of the sum $\rho_s(1-\varepsilon)gz + \Delta p_a$, we obtain for Δp_r

$$\Delta p_r = \rho_s(1-\varepsilon)gz + \Delta p_a + \Delta p_t. \tag{5}$$

Assuming that all the particles entering the lower part of the riser are accelerated there to a velocity $u-u_t$, we can write for Δp_a

$$\Delta p_a = J_s(u-u_t). \tag{6}$$

The quantity Δp_{tr} can easily be determined from the profile of the particle concentration in the transport zone:

$$\Delta p_{tr} = g\int_z^H \rho(h)\,dh. \tag{7}$$

In [10], a generalized equation is obtained for the function $\rho(h)$:

$$\rho(h)/\rho_s = \bar{J}_s(h/H)^{-0.82}. \tag{8}$$

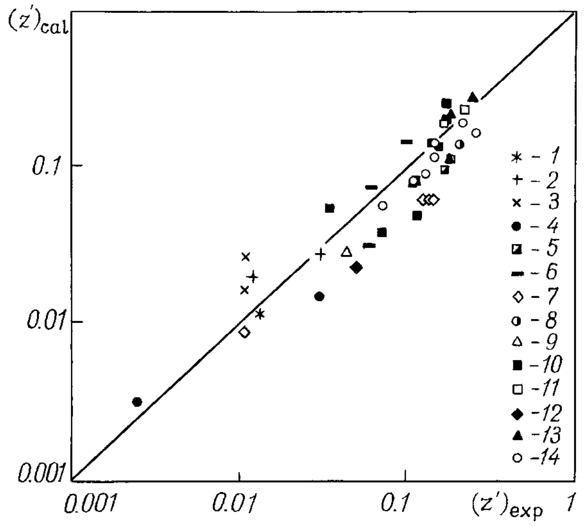

Fig. 2. Height of the fluidized bed in a "chemical reactor"-type CFB: 1) [14]; 2) [16]; 3) [15]; 4) [1]; 5) [17]; 6) [18]; 7) [19]; 8) [20]; 9) [21]; 10) [2]; 11) [3]; 12) [22]; 13) [5]; 14) [23].

With account for (3) and (6)–(8), expression (5) in dimensionless form is

$$0.6\mathrm{Fr}_t^{0.68} - \frac{(1-\varepsilon)\,z'}{\bar{J}_s} - 5.5\,(1-(z')^{0.18}) - \mathrm{Fr}_t = 0\,. \tag{9}$$

It can easily be seen that Eq. (9) allows determination of an important characteristic of a CFB – the height of the fluidized bed at the gas distributor (z). To solve Eq. (9), one must know the porosity of this bed ε. Processing of experimental data [11–13] allowed one to obtain

$$1-\varepsilon = 0.33\,\mathrm{Fr}_t^{-0.045} \tag{10}$$

for $\mathrm{Fr}_t = 7\cdot10^{-4}$–$8\cdot10^{-2}$. With account for (10) the numerical solution of Eq. (9) is approximated with a root-mean-square error of 10% by the following expression for z':

$$z' = 1.25\mathrm{Fr}_t^{-0.8}\,\bar{J}_s^{1.1}\,, \tag{11}$$

where $\mathrm{Fr}_t = 5.8\cdot10^{-7}$–$0.04$ and $\bar{J}_s = 4.4\cdot10^{-7}$–$0.024$. A comparison of results of calculation by (11) with experimental data [1–3, 5, 14–23] (see Table 1) is shown in Fig. 2. It should be noted that due to the high velocity of the gas in the CFB ($u > u_t$, $u \gg u_{\mathrm{mf}}$) the upper boundary of the fluidized bed is not pronounced and, consequently, is determined with low accuracy. In our opinion, this leads to the substantial (~35%) scatter in the experimental points about dependence (11). It is obvious that in this situation formula (11), by reflecting the influence of the main factors, is of special importance, since it allows one to give a reasonable estimate of the quantity z, determined with low accuracy.

"Furnace." In this case, the general dependence of the hydrodynamic characteristic of a CFB on the determining parameters has the form [7]

$$\Gamma' = \varphi\,(H_0/H, \mathrm{Fr}_t, h/H, H/D) = \varphi\left(\frac{\Delta p_{\mathrm{ref}}}{\rho_s\, gH}, \mathrm{Fr}_t, h/H, H/D\right), \tag{12}$$

since $\Delta p_{\mathrm{ref}} = \rho_s g(1-\varepsilon_{\mathrm{mf}})H_0$. The expression for $\Gamma' = \Delta p_r/\rho_s gH$ can be represented as

TABLE 1. Conditions of Experiments on Measuring Δp_r and z

Ref.	H, m	D, m	H/D	u, m/sec	J_s, kg/(m^2·sec)	Δp_{ref}, kPa	d, mm	ρ_s, kg/m^3	Ar	Re$_t$	u_t, m/sec
[1]	9.5	0.25	38.0	6.3	5.1–130.5	–	0.049	2424	10.4	0.52	0.16
[2]	3.56	0.048	74.2	0.84–1.15	2.5–11.7	–	0.077	1628	27.1	1.28	0.25
[3]	15	0.19	78.9	2.6–5.6	105–415	–	0.06	2600	20.5	0.99	0.25
[4]	15.6	0.4	39.0	3, 4	9.3, 23	–	0.2	2600	759.7	21.82	1.64
[5]	8.5	0.411	20.7	2.5–4.5	220–440	–	0.0375	2600	5.0	0.26	0.10
[8]	13.5	1.7×1.7	7.6	1.7–6.3	–	7	0.2, 0.3, 0.4	2600	29, 120, 311	1.4, 4.9, 11	1, 2.3, 3.7
[11–13]	13.5	1.7×1.7	7.6	1.7–6.3	–	7	0.32, 0.44	2600	120, 311	4.9, 10.8	2.3, 3.7
[14]	5.1	0.14	36.4	3	17	–	0.071	1700	22.2	1.06	0.23
[15]	8.5	0.3243	26.2	3.5–4.5	27.6–47.6	–	0.15	2600	320.5	11.08	1.11
[16]	5.1	0.14	36.4	2.6–3	25–26	–	0.06	1700	13.4	0.66	0.17
[17]	2.79	0.05	55.8	1.17–1.3	11.2–11.7	–	0.06	1000	7.9	0.40	0.10
[18]	8.4	0.4	21.0	1.2–3.6	7–40	–	0.085	1500	33.6	1.56	0.28
[19]	6.65	0.205	32.4	1.2–4	8.7, 34	–	0.046	2300	8.2	0.41	0.14
[20]	1.5	0.05	30.0	0.85	2	–	0.1	2500	91.3	3.83	0.58
[21]	4	0.083	48.2	4	43	–	0.21	2650	896.4	24.72	1.77
[22]	13	0.305	42.6	12.2	586	–	0.076	1714	27.5	1.30	0.26
[23]	14	0.2	70.0	3.3–5.1	113–215	–	0.056	1460	9.4	0.47	0.13
[24]	13.5	1.7×1.7	7.6	1.7–5.5	–	2.5–7.7	0.032	1460	120.0	4.90	2.30

TABLE 2. Comparison of Experimental [8] and Calculated (by Eq. (18)) Values of Δp_r for a "Furnace"-Type CFB

u, m/sec	d, mm	Δp_{ref}, Pa	$(\Delta p_r)_{exp}$, Pa	$(\Delta p_r)_{cal}$, Pa
2.7	0.20	7000	9800	9155
2.7	0.32	–	7000	7579
3.5	0.32	–	8000	8700
4.7	0.32	–	9000	10660

$$\frac{\Delta p_r}{\rho_s gH} = \varphi\left(\frac{\Delta p_{ref}}{\rho_s gH}, \mathrm{Fr}_t, H/D\right). \tag{13}$$

The structure of the expression for Δp_r in a "furnace"-type CFB turns out to be simpler:

$$\Delta p_r = \Delta p_{ref} + g\int_{h_0}^{H} \rho(h)\, dh . \tag{14}$$

To calculate the integral in (14), we use a dependence of the particle concentration on the height above the riser that was found in [10]:

$$\rho(h)/\rho_s = 0.053\mathrm{Fr}_t^{0.62}(h/H)^{-0.45} . \tag{15}$$

Formula (15) was obtained by processing experimental data for Δp_{ref} = 7000 Pa. Using the natural assumption that $\rho(h) \sim \Delta p_{ref}$, we can easily generalize (15):

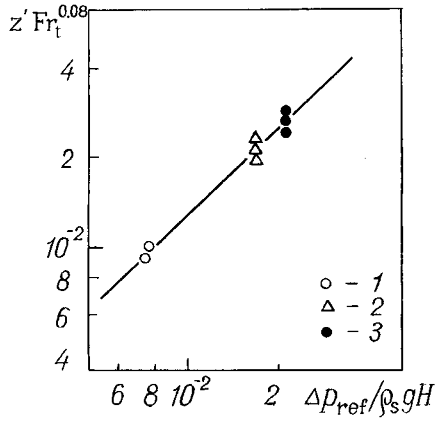

Fig. 3. Height of the fluidized bed in a "furnace"-type CFB: 1) Δp_{ref} = 2500 Pa; 2) 5800; 3) 7700 [24].

$$\rho(h)/\rho_s = 2.6\mathrm{Fr}_t^{0.62}\frac{\Delta p_{ref}}{\rho_s gH}\left(\frac{h}{H}\right)^{-0.45} . \qquad (16)$$

We note dissimilar dependences of $\rho(h)$ on h that follow from formulas (8) and (16). In our opinion, this is explained by the different behavior of the disperse medium in CFB apparatuses with significantly different values of H/D (see above).

Substituting (16) into Eq. (14), we obtain an expression of the type of (13) for determining Δp_r:

$$\frac{\Delta p_r}{\rho_s gH} = \frac{\Delta p_{ref}}{\rho_s gH} + 4.73\mathrm{Fr}_t^{0.62}\left(1 - (h_0/H)^{0.55}\right)\frac{\Delta p_{ref}}{\rho_s gH} . \qquad (17)$$

In more compact form, Eq. (17) is as follows:

$$\frac{\Delta p_r}{\Delta p_{ref}} = 1 + 4.73\mathrm{Fr}_t^{0.62}\left(1 - (h_0/H)^{0.55}\right) . \qquad (18)$$

A comparison of experimental data [8] with results of calculation by (18) is given in Table 2.

Due to the fairly large diameters of the riser in a "furnace"-type CFB, the fluidized bed at the gas distributor is rather pronounced, as a rule, and its height is determined more accurately. Processing of experimental data [24] allowed one to obtain the following simple expression for z:

$$z' = 1.24\frac{\Delta p_{ref}}{\rho_s gH}\mathrm{Fr}_t^{-0.08} . \qquad (19)$$

Equation (19) and experimental data [24] are shown in Fig. 3. The root-mean-square deviation of the experimental points from (19) is 7%. It is easily seen that both (19) and (17) are a particular case of the general expression (12).

A search for the "Rosette stone" – a means of passage from the system of dimensionless groups (1) to system (12) – is of utmost importance in the aspect of correct description of transport processes within the framework of similarity theory. It is obvious that solution of this problem consists in determination of the relation $\bar{J}_s = \bar{J}_s\left(\frac{\Delta p_{ref}}{\rho_s gH}, \mathrm{Fr}_t\right)$, which accomplishes passage from (1) to (12).

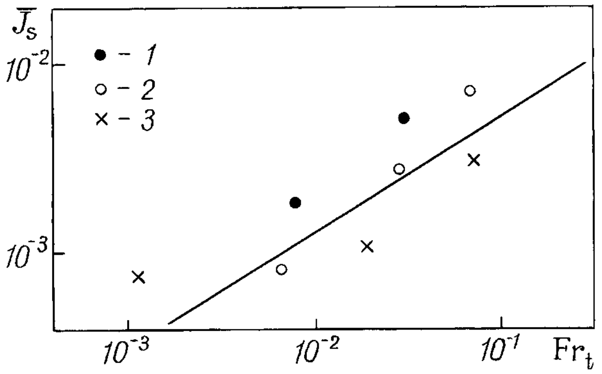

Fig. 4. Mass circulating flux of particles in a "furnace"-type CFB: 1) d = 0.20 mm; 2) 0.32; 3) 0.44 [8].

Equation (9) obtained above allows one to express a dimensionless circulating flux of particles in the following form:

$$\bar{J}_\mathrm{s} = \frac{(1-\varepsilon)\,z'}{0.6\mathrm{Fr_t^{-0.68}} - 5.5\,(1-(z')^{0.18}) - \mathrm{Fr_t}}\,. \tag{20}$$

Substitution of the expressions for $(1-\varepsilon)$ and z' from (10) and (19) into (20) gives the sought relation:

$$\bar{J}_\mathrm{s} = \frac{0.41\,\dfrac{\Delta p_\mathrm{ref}}{\rho_\mathrm{s}\,gH}\,\mathrm{Fr_t^{-0.125}}}{0.6\mathrm{Fr_t^{-0.68}} - 5.5\left(1-\left(\dfrac{\Delta r_\mathrm{ref}}{\rho_\mathrm{s}\,gH}\right)^{0.18}\mathrm{Fr_t^{-0.014}}\right) - \mathrm{Fr_t}}\,. \tag{21}$$

In the region $10^{-3} \le \Delta p_\mathrm{ref}/\rho_\mathrm{s} gH \le 10^{-1}$, $10^{-4} \le \mathrm{Fr_t} \le 4\cdot 10^{-2}$ expression (21) is approximated as follows:

$$\bar{J}_\mathrm{s} = 1.13\,\frac{\Delta p_\mathrm{ref}}{\rho_\mathrm{s}\,gH}\,\mathrm{Fr_t^{0.62}}\,. \tag{22}$$

For the operating conditions of a boiler with a CFB [8] (Δp_ref = 7000 Pa, H = 13.5 m, and ρ_s = 2600 kg/m^3), for which experimental data on J_s are available, expression (22) is simplified:

$$\bar{J}_\mathrm{s} = 0.023\mathrm{Fr_t^{0.62}}\,. \tag{23}$$

Figure 4 presents dependence (23) and experimental points [8]. The rather good agreement between them indicates the validity of the obtained expression (22), which allows passage from the description of a CFB in terms of a "chemical reactor" to a description in terms of a "furnace." Formula (22) is also of great practical importance: it makes it possible to estimate the value of the mass particle flux J_s in freely circulating CFBs ("furnace"), where experimental determination of J_s is, as a rule, fraught with considerable difficulties.

The obtained formulas (3), (11), (18), (19), and (22) have a dimensionless form, are verified over rather wide ranges of variation of the experimental conditions, and can be recommended for use in engineering practice.

NOTATION

$\mathrm{Ar} = \frac{gd^3}{\nu_f^2}\left(\frac{\rho_s}{\rho_f} - 1\right)$, Archimedes number; d, particle diameter; D, riser diameter; $\mathrm{Fr}_t = \frac{(u - u_t)^2}{gH}$, Froude number; g, free-fall acceleration; h_0, upper boundary of the near-grid region with specified resistance Δp_{ref}; h, height above the gas distributor; H, riser height; H_0, height of a hypothetical dense bed with resistance Δp_{ref} ($\Delta p_{\mathrm{ref}} = \rho_s(1 - \varepsilon_{\mathrm{mf}})gH_0$); J_s, specific mass circulating flux of particles; $\overline{J}_s = J_s/\rho_s(u - u_t)$; Δp, head loss; Δp_{ref}, specified pressure drop in the lower part of the riser; $\mathrm{Re}_t = \frac{u_t d}{\nu_f}$; u, velocity of gas filtration; z, height of the fluidized bed in the lower part of the riser; $z' = z/H$; ε, fluidized-bed porosity; ν_f, kinematic viscosity of the gas; ρ, density. Subscripts: a, particle acceleration; cal, calculation; exp, experiment; f, gas; fb, fluidized bed; fr, friction against the riser walls; mf, onset of fluidization; r, riser; s, particles; t, conditions of floating of an individual particle; tr, transport zone.

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